6fi hU`Yg`J YfVfYWX Yb`]b`; Yfg\Y]a .`Gc\b hÒhYh`, -!^À\f][Y`A i hhYf°

9]b'*(! A f][Yf'Ui g'; Yfg\Y]a 'k i fXY'ZØf'XYb'A cfX'Ub'gY]bYf , -! A f][Yb'A i hhYf'ni 'bYi b' A f)[Yb'<UZh'j Yfi fhY]`h"



bUWX 'XYa 'I a [Ub['a]h'dgmWX]gWX '9f_fUb_hYb']a 'FYWX hggmghYaUi Z''

8]Y'8]g_i gg]cb'ØVYf'X]Y'fYW\h']W\Yb'FU\a YbVYX]b[i b[Yb'ZØfHOhi b[gXY`]_hY']b'8Yi hgW\`UbX'\Uh']b'XYb'`YhnhYb'\\Y\fYb'Ub
6YXYi hi b['[Yk cbbYb'"\degVYgcbXYfY']a \dangle \dangle \f'&\f'\\ ('[f]ZZ'XYf
XUa U`][Y'6i bXYg'\f gh]na]b]ghYf'<Y]_c'A UUg'XUg'H\Ya U'Ui Z'i bX
fY[hY'Y]bY'[fi bX'Y[YbXY'FYZcfa 'XYf'ghfUZfYW\h']W\Yb
Bcfa]Yfi b['Ub''8UVY]'ghY``hY'XYf'8Yi hgW\Y'5bk U`hj YfY]b'f85J\text{Y}]bYb'JcfgW\'U['j cfz'XYf'X]Y'6Y[f]ZZY'A cfX'i bX'HchgW\'U[
UVgW\UZZYb'i bX'ghUhhXYggYb'Y]bYb'Y]b\Y]h']W\Yb
HOhi b[ghUhVYghUbX'Y]bZØ\fYb'k c``hY'''9]b'gc'W\Yf'HUhVYghUbX
gc``hY'a]h'Y]bYf': fY]\Y]hgghfUZY'nk]gW\Yb'UW\h'\J\fYb'i bX
'YVYbg`\Ab[`]W\'[YU\bXYh'k YfXYbz'c\bY'[YbUi Y'8]ZZYfYbn]Yfi b[
XYf'GW\i XUb[Ya YggYb\Y]h''8]Yg'nY][hz'XUgg'XYf'fYW\h']W\Y'i bX
[YgY``gW\UZh`]W\Y'I a [Ub['a]h'HOhi b[gXY`]_hYb'_cbh]bi]Yf`]W\

FYWX hg`U[YibX\]ghcf]gWXYDYfgdY_h]jYb

 $\label{eq:converse_sign} 5\text{`V]b'9gYfžY]b'a U* [YV`]WXYf'JYfZUggYf'Y]bYg'FYZcfajcfgWX`U[gjcb'%-, $ž'k]Yg']b'X]YgYa'?cbhYlh'UiZ'X]Y'\]ghcf]gWXY'8]g_igg]cbia X]Y'AcfXeiU`]Z]_Uh]cbYb'\]b'''8]Y'fYWXh`]WXYb <math display="block">FU\aYbVYX]b[ib[Yb'\UVYb'g]WX'gY]h'XYf'5VgWXUZZib[XYfHCXYgghfUZY']b'XYf'6ibXYgfYdiV`]_'8YihgWX`UbX'%-(-'jYfÀbXYfh''8Ug'6ibXYgjYfZUggib[g[Yf]WXh'ibX'XYf'6ibXYg[Yf]WXhg\cZ\UVYb'X]Y'AcfXaYf_aU'Y'fYWXhgghUUh`]WX'Uig[Y`Y[hžiak]``_Øf`]WXY'FYWXhgUbkYbXib[Yb'ni'jYfaY]XYb'''8]Yg'kUf VYgcbXYfg'k]WXh][ž'XU'XYf'ifgdfØb[`]WXY'JcfgWX`U['jcbGhUUhggY_fYhÀf'Fc`UbX':fY]g`Yf'kÀ\fYbX'XYf'BG!NY]h'Y]bY'gc`WXYK]``_Øf'jcf[YgY\Yb'\UhhY''$

5_hi Y``Y`NU\`Yb`VY`Y[Yb`X]Y`8f]b[`]W_Y]h`X]YgYf`8]g_i gg]cb"'*\Df[?]bn][`VYf]W\hYhY`j cb`Y]bYa `F\@W_[Ub[`XYf`H\Dhi b[gXY`]_hY`i a Y]b`J]YfhY``]b`XYb``YhnhYb`%) `*\J\fYb"'; `Y]W\nY]h][`gh]Y[`X]Y`NU\`XYf`\DgUggYb`a]h``YVYbg`Ub[Yf`GhfUZY`V]g`&\$\%'Ui Z`@VYf`&\$\$\$`VY]]bg[YgUa h'), "\$\$\$`GhfUZ[YZUb[YbYb"'\DhYfYggUbhYfk Y]gY`bU\aX]Y`EichY`XYf`kY[Yb`AcfXYq'JYfifhY]`hYb`c\bY'jYfa]bXYfhY

DgmVX]Uhf]gVXY'5gdY_hY'i bX'X]Y': cfYbg]_

8]Y'6YfØW_g]WXh][ib['dgmWX]Uhf]gWXYf'5gdY_hY']b'HÒhib[gXY`]_hYbk]fX'YVYbZU``g'nibY\aYbX']b'XYb':c_ig'[YfØW_h'':cfgWXYf'k]Y

<Ubg!@Xk]['?fÖVYf'VYhfUWXhYb'X]Y'AcfXaYf_aU`Y'Uig

dgmWX]Uhf]gWXYf'G]WXhibX'X]g_ih]YfYb'XUg'GdUbbib[gjYf\À`hb]g

nk]gWXYb'DgmWXcdUh\c`c[]Y'ibX'XYf'cdYfUh]cbU`]g]YfhYb

8]U[bcgh]_']b'XYf'ZcfYbg]gWXYb'DgmWX]Uhf]Y''DUi`'<cZZ'kY]gh'XUfUiZ\]bž'XUgg'X]Y'EiU`]hÀh'_f]a]bU`dc`]nY]`]WXYf'9fa]hh`ib[gUfVY]hiibX

JYfÀbXYfib[Yb'_f]a]bU`dfc[bcgh]gWXYf'F]g]_cZU_hcfYb'YVYbZU``g_f]h]gWX'VYcVUWXhYh'kYfXYb'aØggYbž'ia'dfAjYbh]jYb

;YZU\fYbUVkY\faU»bU\aYb'ni'jYfVYggYfb''

8]Y'fYW\h`]W\Yb'9bhgW\Y]Xi b[Yb']a ': U```XYg'* (!:À\f][Yb'gd]Y[Y`bY]bY'J]Y`nU\`j cb'<YfUi gZcfXYfi b[Yb'k]XYfžX]Y'XUg'XYi hgW\Y
GhfUZfYW\h'i bX'X]Y'XUa]h'j YfVi bXYbYb'[YgY``gW\UZh`]W\Yb'i bX
dgmW\c`c[]gW\Yb': fU[YghY``i b[Yb'VYhfYZZYb''K \A\fYbX'X]Y
; YgY``gW\UZhY]b'A]b]a i a 'Ub'G]W\Yf\Y]h'ZcfXYfhžghY``h'g]W\'X]Y
: fU[Yž'k]Y'a Ub'[`Y]W\nY]h]['XYb'6YXØfZb]ggYb'dgmW\]gW\'_fUb_Yf
H\AhYf'[YfYW\h'k YfXYb'_Ubbž'c\bY'XUg'; Yk U`ha cbcdc``XYg
GhUUhYg'ni '[YZ\A\fXYb'''\=\b'X]YgYa 'GdUbbi b[gZY'X'V'Y]Vh'X]Y
8]g_i gg]cb'\@VYf'X]Y'FYZcfa Yb'XYf'H\Ohi b[gXY`]_hY'fY`Yj Ubh'i bX
bchk YbX]['''9I dYfhYb'ZcfXYfb'cZhž'X]Y'5bXfc\i b['XYf
'YVYbg`Ub[Yb': fY]\Y]hgghfUZY'VY]ni VY\U`hYbž'i a 'XYb
5bZcfXYfi b[Yb'Y]bYf'[YfYW\hYb'i bX'YZZY_h]j Yb'GhfUZj YfZc`[i b[
ni 'YbhgdfYW\Yb''

 $: \emptyset f `k Y] h Y f Z \emptyset \land f Y b X Y ` \pm b Z c f a U h] c b Y b ` g h Y \land Y b ` X] Y ` 5 f h] _ Y ` ` X Y g ` G h Y f b i b X ' j c b ` G d f] b [Y f ` n i f ` J Y f Z \emptyset [i b [" 8] Y g Y ` V Y ` Y i W h Y b ` g c k c \ ` ` X] Y f Y W h`] W X Y b ` U ` g ` U i W ` X] Y ` d g m W \ c ` c [] g W X Y b ` D Y f g d Y _ h] j Y b ` X] Y g Y f _ c a d ` Y I Y b ` H \ Y a U h] _ "$

Cfh	; Yfg\Y]a ž'8Yi hgW\`UbX
EiY``Yb	• k k k "ghYfb"XY
	`]b_"gdf]b[Yf"Wca

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'a Y]b!k]Yb "bYh